A CENTRAL LIMIT THEOREM FOR THE SNR AT THE WIENER FILTER OUTPUT FOR LARGE DIMENSIONAL SIGNALS

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ABSTRACT

Consider the quadratic form $\beta = \mathbf{y}^*(\mathbf{Y}\mathbf{Y}^* + \rho\mathbf{I})^{-1}\mathbf{y}$ where ρ is a positive number, where $\mathbf{y}$ is a random vector and $\mathbf{Y}$ is a $N \times K$ random matrix both having independent elements with different variances, and where $\mathbf{y}$ and $\mathbf{Y}$ are independent. Such quadratic forms represent the Signal to Noise Ratio at the output of the linear Wiener receiver for multi dimensional signals frequently encountered in wireless communications and in array processing. Using well known results of Random Matrix Theory, the quadratic form β can be approximated with a known deterministic real number $\bar{\beta}_K$ in the asymptotic regime where $K \to \infty$ and $K/N \to \alpha > 0$. This paper addresses the problem of convergence of β. More specifically, it is shown here that $\sqrt{K}(\beta - \bar{\beta}_K)$ behaves for large K like a Gaussian random variable which variance is provided.

Index Terms— Antenna Arrays, CDMA, Central Limit Theorem, MC-CDMA, Random Matrix Theory, Wiener Filtering.

1. INTRODUCTION

Consider the N dimensional received signal

$$\mathbf{r} = \mathbf{\Sigma}\mathbf{s} + \mathbf{n}$$

where $\mathbf{s} = [s_0, s_1, \ldots, s_K]^{\mathrm{T}}$ is the transmitted complex vector signal with size $K+1$ satisfying $\mathbb{E}\mathbf{s}\mathbf{s}^* = \mathbf{I}_{K+1}$, matrix $\mathbf{\Sigma}$ represents the "channel" in the wide sense and $\mathbf{n}$ is the independent AWGN with covariance matrix $\mathbb{E}\mathbf{n}\mathbf{n}^* = \rho\mathbf{I}_N > \mathbf{0}$. In this article, we are interested in the performance of the linear Wiener estimate (also called LMMSE for Linear Minimum Mean Squared Error estimate) of signal s_0. Among the various performance indexes, we shall focus on the Signal to Noise Ratio (SNR) which can be expressed as follows: Partition the channel matrix as $\mathbf{\Sigma} = [\mathbf{y}\ \mathbf{Y}]$, then the Wiener estimate $\hat{s}_0$ of s_0 writes $\hat{s}_0 = \mathbf{y}^* \left(\mathbf{\Sigma}\mathbf{\Sigma}^* + \rho\mathbf{I}_N\right)^{-1} \mathbf{r}$ and the associated SNR β_K is given by:

$$\beta_K = \mathbf{y}^* \left(\mathbf{Y}\mathbf{Y}^* + \rho\mathbf{I}_N\right)^{-1} \mathbf{y} \,.$$

A popular tool to address this problem, widely used in multidimensional signal processing and communication engineering, is represented by the theory of Large Random Matrices: Assume that $\mathbf{\Sigma}$ is random (in this case, β_K becomes a conditional SNR) and let $N \to \infty$ with $K/N \to \alpha > 0$ (denoted in the sequel by "$K \to \infty$" for short). As amply shown in the literature, there are many statistical models related to $\mathbf{\Sigma}$ for which there exists a deterministic sequence $\bar{\beta}_K$ such that $\beta_K - \bar{\beta}_K \to 0$ almost surely (a.s.); this approximation is generally defined as the solution of an implicit equation. Beyond the convergence of the SNR, a natural practical and theoretical problem concerns the study of its fluctuations (think for instance to the outage probability evaluations). Despite its interest, there are very few related articles in the literature. In this paper, we provide a Central Limit Theorem (CLT) for β_K as $K \to \infty$ for a general model of matrix $\mathbf{\Sigma}$: Assume that the $N \times (K+1)$ matrix $\mathbf{\Sigma}$ is given by:

$$\mathbf{\Sigma} = \frac{1}{\sqrt{K}} \left[\sigma_{nk} W_{nk}\right]_{n=1,k=0}^{N,K} \tag{1}$$

where $(\sigma_{nk}^2;\ 1 \le n \le N;\ 0 \le k \ldots, K)$ is a sequence of real numbers called a variance profile and where the complex random variables W_{nk} are independent and identically distributed (i.i.d.) with $\mathbb{E}W_{nk} = 0$, $\mathbb{E}W_{nk}^2 = 0$, and $\mathbb{E}|W_{nk}|^2 = 1$. In this case, the quadratic form β_K is given by:

$$\beta_K = \frac{1}{K} \mathbf{w}_0^* \mathbf{D}_0^{1/2} \left(\mathbf{Y}\mathbf{Y}^* + \rho\mathbf{I}_N\right)^{-1} \mathbf{D}_0^{1/2} \mathbf{w}_0 \tag{2}$$

where $\mathbf{w}_0 = [W_{10}, W_{20}, \ldots, W_{N0}]^{\mathrm{T}}$ and $\mathbf{D}_0$ is the $N \times N$ diagonal nonnegative matrix $\mathbf{D}_0 = \mathrm{diag}(\sigma_{10}^2, \ldots, \sigma_{N0}^2)$. An important special case that we shall describe carefully in the sequel is when the variance profile is *separable*, i.e $\sigma_{nk}^2 = d_n \tilde{d}_k$.

Among the many applications of the general model (1), let us mention:

- Multiple antenna transmissions with $K+1$ antennas at the transmission side and N antennas at the reception side. Here we consider the transmission model $\mathbf{r} = \mathbf{\Xi}\mathbf{s} + \mathbf{n}$ where $\mathbf{\Xi} = \frac{1}{\sqrt{K}}\mathbf{H}\mathbf{P}^{1/2}$, matrix $\mathbf{H}$ is a $N \times (K+1)$ random matrix with complex Gaussian elements representing the radio channel and $\mathbf{P} = \mathrm{diag}(p_0, \ldots, p_K)$ is the (deterministic) matrix of the powers given to the different users. Write $\mathbf{H} = [\mathbf{h}_0 \ \cdots \ \mathbf{h}_K]$, and assume the columns $\mathbf{h}_k$ are independent, which is realistic when the transmitters are distant one from another. Let $\mathbf{C}_k$ be the covariance matrix $\mathbf{C}_k = \mathbb{E}\mathbf{h}_k\mathbf{h}_k^*$ and let $\mathbf{C}_k = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k$ be a spectral decomposition of $\mathbf{C}_k$ where $\mathbf{\Lambda}_k = \mathrm{diag}((\lambda_{nk})_{n=1,\ldots,N})$ is the matrix of eigenvalues. If the eigenvector matrices $\mathbf{U}_0, \ldots, \mathbf{U}_K$ are all equal (note that sometimes they are all identified with the Fourier $N \times N$ matrix [1]), then one can show that matrix $\mathbf{\Xi}$ introduced above can be replaced with matrix $\mathbf{\Sigma}$ of Model (1) where the W_{nk} are standard Gaussian iid and $\sigma_{nk}^2 = \lambda_{nk}p_k$. In certain situations it is furthermore assumed that $\mathbf{\Lambda}_0 = \cdots = \mathbf{\Lambda}_K = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$: this is the well known Kronecker model with correlations at reception. In our setting this model is accounted for by the separable variance profile case $\sigma_{nk}^2 = \lambda_n p_k$.

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- CDMA transmissions on flat fading channels. Here N is the spreading factor, $K+1$ is the number of users, and

$$\mathbf{\Sigma} = \mathbf{W}\mathbf{P}^{1/2} \tag{3}$$

where $\mathbf{W}$ is the $N \times (K+1)$ signature matrix assumed here to have random i.i.d. elements with mean zero, variance $1/N$ and where $\mathbf{P} = \mathrm{diag}(p_0, \ldots, p_K)$ is the users powers matrix. In this case, the variance profile is separable with $d_n = 1$ and $\tilde{d}_k = \frac{K}{N} p_k$.

- MC-CDMA transmissions on frequency selective channels. In the uplink, the matrix $\mathbf{\Sigma}$ is written:

$$\mathbf{\Sigma} = [\mathbf{H}_0 \mathbf{w}_0 \ \cdots \ \mathbf{H}_{K+1} \mathbf{w}_{K+1}]$$

where $\mathbf{H}_k = \mathrm{diag}(h_k(\exp(2\imath\pi(n-1)/N))_{n=1,\ldots,N})$ is the radio channel matrix of user k in the discrete Fourier domain and $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_K]$ is the $N \times (K+1)$ signature matrix with iid elements as in the CDMA case above. Modeling this time the channels transfer functions as deterministic functions, we have $\sigma^2_{nk} = \frac{K}{N} |h_k(\exp(2\imath\pi(n-1)/N))|^2$. In the downlink, we have

$$\mathbf{\Sigma} = \mathbf{H}\mathbf{W}\mathbf{P}^{1/2} \tag{4}$$

where $\mathbf{H} = \mathrm{diag}(h(\exp(2\imath\pi(n-1)/N))_{n=1,\ldots,N})$ is the radio channel matrix in the discrete Fourier domain, the $N \times (K+1)$ signature matrix $\mathbf{W}$ is as above, and $\mathbf{P} = \mathrm{diag}(p_0, \ldots, p_K)$ is the matrix of the powers given to the different users. Model (4) coincides with the separable variance profile case with $d_n = \frac{K}{N} |h(\exp(2\imath\pi(n-1)/N))|^2$ and $d_k = p_k$.

About the Literature. In the communication engineering literature, the CLT for the quadratic forms has been considered probably for the first time in [2], where the authors consider the case where $\mathbf{\Sigma}$ is a matrix with i.i.d. elements. Their results are based on [3] where the asymptotic behaviour of the eigenvectors of $\mathbf{\Sigma}\mathbf{\Sigma}^*$ is described. Recently, [4] considered the more general CDMA Model (3). The model considered in this paper includes the models of [2] and [4] as special cases. The approach used here to establish the CLT is powerful yet simple. It is based on the representation of β_K as the sum of a martingale difference sequence and the use of the CLT for martingales [5].

This paper is organized as follows. In Section 2 we recall the first order results that describe the limiting behaviour of β_K. The CLT for β_K is stated in Section 3. A sketch of proof for the CLT is presented in Section 4. Finally, we provide simulations in Section 5.

2. SNR DETERMINISTIC APPROXIMATION

Let us begin with a definition and some notations. We say that a complex function $t(z)$ belongs to class $\mathcal{S}$ if $t(z)$ is analytical in the upper half plane $\mathbb{C}_+ = \{z \in \mathbb{C} \ : \ \mathrm{im}(z) > 0\}$, if $t(z) \in \mathbb{C}_+$ for all $z \in \mathbb{C}_+$ and if $\mathrm{im}(z)|t(z)|$ is bounded over $\mathbb{C}_+$. We introduce the diagonal matrices

$$\begin{aligned} \mathbf{D}_k &= \mathrm{diag}(\sigma^2_{1k}, \ldots, \sigma^2_{Nk}), \quad k = 1, \ldots, K \\ \widetilde{\mathbf{D}}_n &= \mathrm{diag}(\sigma^2_{n1}, \ldots, \sigma^2_{nK}), \quad n = 1, \ldots, N \end{aligned}$$

and the diagonal matrix functions

$$\begin{aligned} \mathbf{T}(z) &= \mathrm{diag}(t_1(z), \ldots, t_N(z)) \\ \widetilde{\mathbf{T}}(z) &= \mathrm{diag}(\tilde{t}_1(z), \ldots, \tilde{t}_K(z)) \end{aligned}$$

that are specified by the following proposition:

Proposition 1 *([6, 7]) The system of $N+K$ functional equations*

$$\left\{ \begin{aligned} t_n(z) &= \frac{-1}{z\left(1 + \frac{1}{K}\mathrm{tr}(\widetilde{\mathbf{D}}_n \widetilde{\mathbf{T}}(z))\right)}, \quad 1 \le n \le N \\ \tilde{t}_k(z) &= \frac{-1}{z\left(1 + \frac{1}{K}\mathrm{tr}(\mathbf{D}_k \mathbf{T}(z))\right)}, \quad 1 \le k \le K \end{aligned} \right.$$

has a unique solution $(\mathbf{T}, \widetilde{\mathbf{T}})$ among the diagonal matrices for which the t_n and the $\tilde{t}_k$ belong to class $\mathcal{S}$. Functions $t_n(z)$ and $\tilde{t}_k(z)$ so defined admit analytical continuations over $\mathbb{C} - [0, \infty)$.

In the separable case, we have $\mathbf{D}_k = \tilde{d}_k \mathbf{D}$ and $\widetilde{\mathbf{D}}_n = d_n \widetilde{\mathbf{D}}$ where $\mathbf{D} = \mathrm{diag}(d_1, \ldots, d_N)$ and $\widetilde{\mathbf{D}} = \mathrm{diag}(\tilde{d}_1, \ldots, \tilde{d}_K)$. In this case, the system described above simplifies to a system of two equations:

Proposition 2 *The system of two functional equations*

$$\left\{ \begin{aligned} \delta(z) &= \frac{1}{K}\mathrm{tr}\left(\mathbf{D}\left(-z(\mathbf{I}_N + \tilde{\delta}(z)\mathbf{D})\right)^{-1}\right) \\ \tilde{\delta}(z) &= \frac{1}{K}\mathrm{tr}\left(\widetilde{\mathbf{D}}\left(-z(\mathbf{I}_K + \delta(z)\widetilde{\mathbf{D}})\right)^{-1}\right) \end{aligned} \right. \tag{5}$$

admits a unique solution $(\delta, \tilde{\delta}) \in \mathcal{S}^2$. Moreover, letting $z = -\rho \in (-\infty, 0)$, the system admits a unique pointwise solution $(\delta(-\rho), \tilde{\delta}(-\rho))$ such that $\delta(-\rho) > 0$, $\tilde{\delta}(-\rho) > 0$.

In this particular case, the matrix functions $\mathbf{T}$ and $\widetilde{\mathbf{T}}$ defined by Proposition 1 are given by $\mathbf{T} = -\frac{1}{z}(\mathbf{I} + \tilde{\delta}\mathbf{D})^{-1}$ and $\widetilde{\mathbf{T}} = -\frac{1}{z}(\mathbf{I} + \delta\widetilde{\mathbf{D}})^{-1}$. The asymptotic behaviour of β_N is characterized by the following theorem:

Theorem 1 *([6, 8, 7]) Let $\bar{\beta}_K = \dfrac{1}{K}\mathrm{tr}\mathbf{D}_0\mathbf{T}(-\rho)$ where $\mathbf{T}$ is given by Proposition 1. Then*

$$\beta_K - \bar{\beta}_K \xrightarrow[K\to\infty]{} 0 \quad \textit{almost surely.}$$

Remark 1 *In matrix model (1), one sometimes assumes that the variance profile σ^2_{nk} is obtained from the samples of a continuous nonnegative function $\pi(x, y)$ defined on $[0,1]^2$ at points $(n/N, k/(K+1))$, i.e. $\sigma^2_{nk} = \pi(n/N, k/(K+1))$. In this particular case, the sequences $\bar{\beta}_K$ and δ_K defined in Theorem 1 above (and also Corollary 1 below) converge to limits that are solutions of integral equations (see for instance [8, 9]).*

In the separable case, $\mathbf{D}_0 = \tilde{d}_0 \mathbf{D}$ hence we have

Corollary 1 *([8, 9]) Assume the separable case $\sigma^2_{nk} = d_n \tilde{d}_k$. Then*

$$\frac{\beta_K}{\tilde{d}_0} - \delta_K \xrightarrow[K\to\infty]{} 0 \qquad \textit{a.s.}$$

where $\delta_K = \delta$ with $(\delta, \tilde{\delta})$ being the solution of System (5) at $z = -\rho$.

Remark 2 *(see also Corollary 2 below) In the separable case, $\beta_K/\tilde{d}_0$ often represents the SNR of user 0 normalized to this user's power. Therefore, we can naturally interpret the approximation δ_K as an asymptotic normalized SNR. This approximation, as well as the asymptotic variance of the normalized SNR $\beta_K/\tilde{d}_0$ defined in Corollary 2 is the same for all users.*

3. SNR FLUCTUATIONS: THE CLT

We now come to the main result of this paper, which holds true under some slight technical assumptions:

Theorem 2 *Let $\mathbf{A}$ and $\boldsymbol{\Delta}$ be the $K \times K$ matrices*

$$\mathbf{A} = \left[\frac{1}{K} \frac{\frac{1}{K}\mathrm{tr}\mathbf{D}_\ell \mathbf{D}_m \mathbf{T}(-\rho)^2}{\left(1 + \frac{1}{K}\mathrm{tr}\mathbf{D}_\ell \mathbf{T}(-\rho)\right)^2} \right]_{l,m=1}^{K} \quad \text{and}$$

$$\boldsymbol{\Delta} = \mathrm{diag}\left(\left(1 + \frac{1}{K}\mathrm{tr}\mathbf{D}_l \mathbf{T}(-\rho)\right)^2_{l=1,\dots,K} \right)$$

where $\mathbf{T}$ is defined by Proposition 1. Let $\mathbf{g}$ be the $K \times 1$ vector

$$\mathbf{g} = \left[\frac{1}{K}\mathrm{tr}\mathbf{D}_0\mathbf{D}_1\mathbf{T}(-\rho)^2, \cdots, \frac{1}{K}\mathrm{tr}\mathbf{D}_0\mathbf{D}_K\mathbf{T}(-\rho)^2 \right]^{\mathrm{T}}$$

Then the following hold true :

1) *The sequence of real numbers*

$$\Theta_K^2 = (\mathbb{E}|W_{10}|^4 - 1)\frac{1}{K}\mathrm{tr}\mathbf{D}_0^2\mathbf{T}^2 + \frac{1}{K}\mathbf{g}^{\mathrm{T}}(\mathbf{I}_K - \mathbf{A})^{-1}\boldsymbol{\Delta}^{-1}\mathbf{g} \quad (6)$$

is well defined and furthermore

$$0 < \liminf_K \Theta_K^2 \le \limsup_K \Theta_K^2 < \infty$$

2) *The sequence β_K satisfies*

$$\sqrt{K}\frac{\beta_K - \bar{\beta}_K}{\Theta_K} \xrightarrow[K\to\infty]{} \mathcal{N}(0,1)$$

in distribution where $\bar{\beta}_K$ is defined in the statement of Theorem 1.

In the separable case, one can show that $\Theta_K^2 = \tilde{d}_0^2\Omega_K^2$ where Ω_K^2 is given by the following corollary:

Corollary 2 *Assume the separable case $\sigma_{nk}^2 = d_n\tilde{d}_k$. Let $\gamma = \frac{1}{K}\mathrm{tr}\mathbf{D}^2\mathbf{T}^2$ and $\tilde{\gamma} = \frac{1}{K}\mathrm{tr}\widetilde{\mathbf{D}}^2\widetilde{\mathbf{T}}^2$. The sequence*

$$\Omega_K^2 = \gamma\left(\left(\mathbb{E}|W_{10}|^4 - 1\right) + \frac{\rho^2\gamma\tilde{\gamma}}{1 - \rho^2\gamma\tilde{\gamma}} \right)$$

satisfies $0 < \liminf_K \Omega_K^2 \le \limsup_K \Omega_K^2 < \infty$, and

$$\sqrt{K}\frac{\beta_K/\tilde{d}_0 - \delta_K}{\Omega_K} \xrightarrow[K\to\infty]{} \mathcal{N}(0,1)$$

in distribution.

Remark 3 *These results show in particular that the asymptotic variance Θ_K^2 is minimum with respect to the distribution of the W_{nk} when $|W_{nk}| = 1$ with probability one. In the context of CDMA and MC-CDMA, this will be the case when the signature matrix elements have their values in a PSK constellation.*

4. SKETCH OF PROOF

Let $\mathbf{Q}$ be the $N \times N$ matrix $\mathbf{Q} = (\mathbf{Y}\mathbf{Y}^* + \rho\mathbf{I}_N)^{-1}$. Recall that the deterministic approximation of β_K is $\bar{\beta}_K = \frac{1}{K}\mathrm{tr}\mathbf{D}_0\mathbf{T}$. Getting back to Equation (2), we can write

$$\begin{aligned} \sqrt{K}(\beta_K - \bar{\beta}_K) &= \frac{1}{\sqrt{K}}\left(\mathbf{w}_0^*\mathbf{D}_0^{1/2}\mathbf{Q}\mathbf{D}_0^{1/2}\mathbf{w}_0 - \mathrm{tr}\mathbf{D}_0\mathbf{Q} \right) \\ &\quad + \frac{1}{\sqrt{K}}\mathrm{tr}\mathbf{D}_0\left(\mathbf{Q} - \mathbf{T}\right) \\ &\stackrel{\mathrm{def}}{=} \xi_K + \chi_K \end{aligned}$$

It can be shown [10] that $\mathbb{E}\chi_K^2 = \mathcal{O}(1/K)$. On the other hand, by using the independence of $\mathbf{w}_0$ and $\mathbf{Q}$ and the fact that the elements of $\mathbf{w}_0$ are i.i.d., one can easily show that $\mathbb{E}\xi_K^2 = \mathcal{O}(1)$ as $K \to \infty$. As a consequence, the asymptotic behaviour of $\sqrt{K}(\beta_K - \bar{\beta}_K)$ is given by ξ_K. Denote by $\mathbb{E}_n$ the conditional expectation $\mathbb{E}_n[.] = \mathbb{E}[.\|W_{n,0}, W_{n+1,0}, \dots, W_{N,0}, \mathbf{Y}]$. Put $\mathbb{E}_{N+1}[.] = \mathbb{E}[.\|\mathbf{Y}]$ and note that $\mathbb{E}_{N+1}\mathbf{w}_0^*\mathbf{D}_0^{1/2}\mathbf{Q}\mathbf{D}_0^{1/2}\mathbf{w}_0 = \mathrm{tr}\mathbf{D}_0\mathbf{Q}$. With these notations at hand, we have

$$\xi_K = \sum_{n=1}^{N}(\mathbb{E}_n - \mathbb{E}_{n+1})\frac{\mathbf{w}_0^*\mathbf{D}_0^{1/2}\mathbf{Q}\mathbf{D}_0^{1/2}\mathbf{w}_0}{\sqrt{K}} \stackrel{\mathrm{def}}{=} \sum_{n=1}^{N} Z_n\ .$$

The sequence Z_n is readily a martingale difference sequence with respect to the increasing sequence of $\sigma-$fields $\sigma(\mathbf{Y}), \sigma(W_{N,0}, \mathbf{Y})), \dots, \sigma(W_{1,0}, \dots, W_{N,0}, \mathbf{Y})$. The asymptotic behaviour of ξ_K (convergence in distribution toward a Gaussian r.v. and derivation of the variance Θ_K^2) can be characterized with the help of the CLT for martingales [5, Ch. 35].

5. SIMULATIONS

In this section, the accuracy of the Gaussian approximation is verified by simulation. We consider an MC-CDMA transmission in the uplink direction. The base station detects the symbols of a given user in the presence of K interfering users. We assume that the discrete channel impulse response of each user consists in $L = 5$ iid Gaussian coefficients with variance $1/L$. All impulse responses are known to the base station.
In this case, $\boldsymbol{\Sigma}$ is given by:

$$\boldsymbol{\Sigma} = [\sqrt{p_0}\mathbf{H}_0\mathbf{w}_0 \ \cdots \ \sqrt{p_{K+1}}\mathbf{H}_{K+1}\mathbf{w}_{K+1}]$$

where

- $\mathbf{H}_k = \mathrm{diag}(h_k(\exp(2\imath\pi(n-1)/, N))_{n=1,\dots,N})$ is the channel matrix of user k in the frequency domain,
- p_k is the amount of power allocated to user k,
- $\boldsymbol{w}_k$ are assumed to belong to QPSK constellation with mean zero and variance $1/N$.

In this case, $\sigma_{n,k}^2$ is given by:

$$\sigma_{n,k}^2 = \frac{Kp_k}{N}|h_k\left(\exp\left(2i\pi(n-1)/N\right)\right)|^2$$

We denote by P the power given to the user of interest. The other users are arranged into 5 classes according to their powers. The power of each class as well as the proportion of users within this class are given in table 1.

Figure 1 shows the histogram of $\sqrt{K}(\beta_K - \bar{\beta}_K)$ for $N = 16$ and $N = 64$. We note that as it was predicted by our derived results, the

Table 1. Power and proportion of each user class

class	1	2	3	4	5
Power	P	$2P$	$4P$	$8P$	$16P$
Proportion	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$

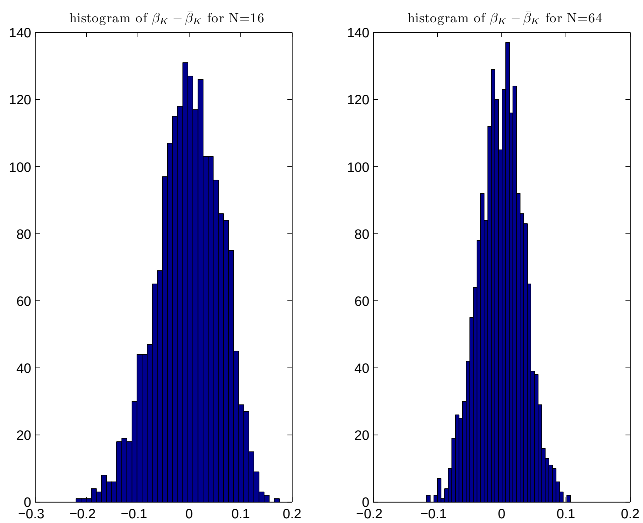

Fig. 1. Histogram of $\sqrt{K}(\beta_K - \bar{\beta}_K)$ for $N = 16$ and $N = 64$.

histogram of $\sqrt{K}(\beta_K - \bar{\beta}_K)$ is similar to that of a Gaussian random variable. In Figure 2 the measured second moment of $\beta_K - \bar{\beta}_K$ is compared with Θ_K^2/K. We note that convergence is reached even for $K = 8$.

6. CONCLUSION

The Gaussian character of the SNR at the output of the Wiener receiver for a class of large dimensional signals described by a random transmission model has been established theoretically and verified by simulation.

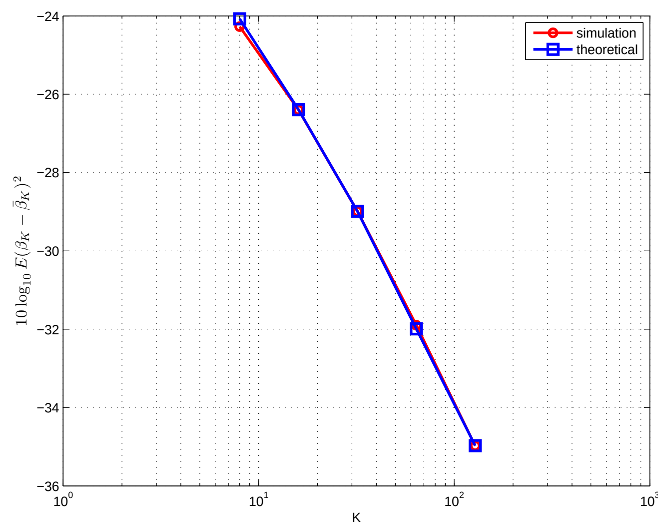

Fig. 2. Second moment of $\beta_K - \bar{\beta}_K$

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